Open Letter to the MILC Collaboration: Why Does the Coarsest Lattice Spacing Stop Just Below the Proton's Compton Wavelength?

Yevhenii Stakhov · ORCID 0009-0005-2585-6432 · Open letter · 2026 · Version 1 (2026-10-13)

Summary

The coarsest ensembles used by the MILC collaboration for precision light-hadron physics have had a lattice spacing of about 0.18 fm for more than twenty years, while the finest end of the sequence has moved steadily down (0.15, 0.12, 0.09, 0.06, ≈0.042, 0.03 fm). The proton's reduced Compton wavelength is $\lambda\!\!\!^{-}_p = \hbar/(m_p c) = 0.21031$ fm. This letter asks three questions about the position of that upper edge. It makes no claim that lattice QCD results are wrong; it asks for the reasoning behind a limit that, as far as the author could find, is nowhere stated explicitly.

Dear colleagues,

I write as the author of a research program on the discrete structure of space (Theory of Space, TOS), in which the proton's reduced Compton wavelength plays a distinguished role as a length scale (Stakhov 2026, Zenodo preprints listed below). While studying your published ensembles I noticed a feature I could not find discussed anywhere, and I would be grateful for your view.

The observation. In lattice units a hadron mass m enters as the dimensionless product $a{\cdot}m$, and the usual requirement $a{\cdot}m \ll 1$ is, written in physical units, $a \ll \hbar/(mc)$ — the reduced Compton wavelength of that hadron. For the proton this gives $a \ll 0.21$ fm. The upper edge of the MILC sequence of ensembles with physical or near-physical light quarks has stood at ≈0.18 fm (86% of $\lambda\!\!\!^{-}_p$) since the early asqtad program, and the RBC/UKQCD domain-wall ensembles also stop near 0.19 fm ($a^{-1} \approx 1.02$ GeV). The ensemble labels form an approximately arithmetic progression of step 0.03 fm at the coarse end (0.18, 0.15, 0.12, 0.09); the next label above 0.18 would be 0.21 fm, i.e. $\lambda\!\!\!^{-}_p$ itself.

What we understand already. We are aware of the standard reasons for keeping a below the hadronic scale, $a^{-1} \gtrsim 1$ GeV; of the growth of taste splitting of staggered quarks on coarse lattices; and of the fact that the ensemble labels are rounded and the spacing is set via r_1, f_π or the Ω^- mass. We note, however, that 1 GeV is essentially the proton mass (0.938 GeV), so the conventional "hadronic scale" coincides with $\lambda\!\!\!^{-}_p$ to within rounding. We also note that heavier hadrons (Ω^- with $\lambda\!\!\!^{-} \approx 0.12$ fm, D mesons with $\lambda\!\!\!^{-} \approx 0.105$ fm) are routinely computed on lattices coarser than their own Compton wavelengths: the hadron's own $\lambda\!\!\!^{-}$ is not, by the usual reasoning, a constraint. For heavy quarks the analogous condition $a{\cdot}m_b < 1$ is stated openly in the literature (e.g. hep-ph/0407221, hep-lat/9705034) and is the reason for effective-theory methods and for the finest HISQ ensembles. For the proton, whose mass is an output rather than an input of the simulation, we found no corresponding statement.

Our questions.

1. Why does the upper limit of the lattice spacing of the precision ensembles lie just below the proton's reduced Compton wavelength — 0.18 fm against $\lambda\!\!\!^{-}_p$ = 0.21031 fm? Is this limit chosen with the proton (or the 1 GeV scale) in mind, or does it follow from another criterion?

2. The coarse end of the ensemble labels runs at intervals of 0.03 fm; the next label above 0.18 fm would be 0.21 fm. What stopped the sequence before it?

3. Have light-hadron spectra been computed, even in exploratory runs, at spacings above 0.21 fm with modern improved actions? If so, how do the discretization errors of the nucleon mass behave as the spacing approaches 0.21 fm — smoothly, as $(a\Lambda)^2$, or with signs of a threshold? Is the upper edge set mainly by taste splitting, or by something else?

A short answer to any of these — including "this is the 1 GeV rule of thumb and nothing more" — would be valuable, and we would report it as such.

On publication. This letter is published openly, with a dated DOI, at theoryof.space and on Zenodo. Any reply will be published in full and without editing, but only with your explicit consent; without consent it will be summarized in my own words and attributed as private correspondence. Replies can be sent to stakhoff@gmail.com.

With respect and gratitude for twenty years of publicly documented ensembles,

Yevhenii Stakhov

Background

The full argument, including the comparison with b-quark Compton wavelengths ($\lambda\!\!\!^{-}_b$ = 0.041–0.047 fm, depending on the mass scheme) and the ≈0.042 fm ensembles, is given in Appendix A1.4 of *Theory of Space, Volume 1: Structure*. The book, an interactive model and the open laboratory: **https://theoryof.space**

References

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