Open Letter to Ole L. Trinhammer and Henrik G. Bohr: The Integer 4 in πr_p = 2λ_p

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Summary

Trinhammer and Bohr (EPL 128, 21001, 2019) derived the proton charge radius from the topology of a wavefunction on the compact configuration space U(3): $\pi r_p = 2\lambda_p$, with λ_p the full Compton wavelength. Written with the reduced Compton wavelength $\lambda\!\!\!^{-}_p = \lambda_p/2\pi$, the same relation reads $r_p = 4\lambda\!\!\!^{-}_p$ — an integer that does not appear in their paper. This letter acknowledges their priority, explains how the integer was found independently in a discrete-lattice model, and asks three questions about its meaning in their framework.

Dear Dr. Trinhammer, dear Dr. Bohr,

I write to thank you and to ask about one number in your 2019 paper [1].

Priority. Your relation $\pi r_p = 2\lambda_p$ gives $r_p = 0.841235641(10)$ fm and, as you state, does not depend on any approximation. In my preprint series [3–5] I reached the relation $r_p = 4\cdot\hbar/(m_p c)$ by a different route — from a discrete tetrahedral-octahedral lattice counted from the Planck length, in which the proton charge radius and the proton's reduced Compton wavelength sit two doubling steps apart, so that their ratio is $2^2 = 4$. Only afterwards did I find your paper. The two relations are algebraically identical: with $\lambda_p = 2\pi\lambda\!\!\!^{-}_p$,

$$\pi r_p = 2\cdot(2\pi\lambda\!\!\!^{-}_p) \Rightarrow r_p = 4\lambda\!\!\!^{-}_p.$$

Priority for the relation belongs to your 2019 work, and my preprints and book say so explicitly. What is new on my side is only the isolation of the integer and its geometric reading. Against the CODATA 2022 radius [6], 0.84075(64) fm, the ratio $Cpr = r_p/\lambda\!\!\!^{-}_p = 3.9977 \pm 0.0030$ lies 0.76σ from 4.

Why the four was easy to miss. In your notation the 2π of the compact torus is visible and the four is not: $r_p/\lambda_p = 2/\pi$, and only on passing to $\lambda\!\!\!^{-}_p$ does the factor 2π cancel and leave a bare integer. Haug and Spavieri (2022) [2] remarked in one sentence that the proton radius is "almost exactly four times" its reduced Compton length, citing your paper; I found no other discussion of it.

My questions.

1. Did you consider writing your result in terms of $\lambda\!\!\!^{-}_p$, and does the integer 4 have a meaning of its own inside the U(3) framework — for example, as the square of the factor 2 that comes from the fractional Bloch phase?

2. Your derivation involves a period doubling of the proton wavefunction (connected with the Higgs mechanism in neutron decay). In the lattice picture the four arises as two successive doublings of length ($2^2 = 4$). Do you see any relation between your period doubling and a doubling of scale, or is the resemblance only verbal?

3. Since 2019 the experimental picture has moved: CODATA 2022 recommends 0.84075(64) fm, and the 2026 atomic-hydrogen measurements give 0.8406(15) fm (MPQ) and 0.8433(31) fm (Colorado State). How do you see the status of your prediction today, and has the framework produced further predictions — for the neutron or other baryons — that could be compared with the integer reading?

On publication. This letter is published openly, with a dated DOI, at theoryof.space and on Zenodo. Your reply, if you send one, will be published in full and without editing only with your explicit consent; otherwise it will be summarized in my own words and attributed as private correspondence. Replies can be sent to stakhoff@gmail.com.

With respect and gratitude for an independent derivation that made the four impossible to dismiss as a coincidence of one model,

Yevhenii Stakhov

Background

The comparison is set out in the Theory of Space book series, Volume 1 (*Structure*), §5.1 and Appendix A5.1. Book, interactive model of the lattice and the open laboratory: **https://theoryof.space**

References

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